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Let’s call the cut set asand the maximum cut as . A cut edge is an edge whose endpoints belong to different sets A and B, while an edge is not a cut edge if its endpoints are both in A, or both in B. Let. Since each node is uniformly assigned at random 0 or 1, it means that

. From this identity, we can calculate:

. Similar for other identities

* 

By definition, we have: 

*  (proven)

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By the definition, we can find the expectation of  as follows:  (since each case occurs equally likely)

On the other hand, we have 

* (linearity of expectation)

Now, let’s define the probability of success as follows:

. From this,



* 
* . Because the probability of success is higher than failure, it follows that:
*  (proven)

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From part (b), we observe that 

*  (from linearity of expectation)

And since  =>  (proven)

Therefore, the expected output of the algorithm isa 2-approximation for maximum cut

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The Markov’s inquality is , where is some constant factor. Let’s call Y a random variable of the number of edges not in the cut. => 

*  => 
*  => 

Let , the inequality becomes:

*  (due to linearity of expectation)

From previous parts, we have 

* , for  (proven)